

Tides

I. General Considerations

A. Introduction

- Written accounts of the tides have existed since 450 B.C. when Herodotus concluded that the tides of the Mediterranean Sea were related to the motion of the moon. In fact, tides are generated by the gravitational attraction of the sun and moon on the ocean masses, affecting every particle of water, from the surface to the depths of the ocean basins.
- It wasn't until Newton (published in 1686) that we had an accurate mathematical description of the tides

B. Review:

- Tides are the ultimate in shallow water waves, since their wavelengths can be thousands of kilometers and their heights can be more than 15 meters.
- Tides are measured as the water height above average water height. They are most noticeable at the coasts, and they affect
- many activities (commercial and recreational) in harbors; in harvesting food (mussels, clams); in navigation; etc. They are of
- particular importance for navigation safety.
- The mean low tide is also known as the *tidal datum*. For navigation purposes this is extremely important to know accurately, and it is printed on navigation charts.

II. Generating Tides

A. Forces

- Tides are generated by a combination of gravity and the motion among Earth, the moon and the sun. According to Newton's Law of Gravitation (published in 1686) every particle with mass in the universe attracts every other particle with mass.

$$\textit{Gravitation force} = G \times m_1 \times m_2 / r_{12}^2.$$

- The gravitation force is directly proportional to the product of the 2 masses and inversely proportional to the square of the distance between them.
- G is the universal gravitational constant, m_1 and m_2 are the masses, and r_{12} is the distance between the masses.
- The *tide-generating force* is proportional to the cube of the distance rather than the square.
- Like gravity, the tide generating force is proportional to the mass of the particles as follows:

$$\textit{Tide-generating Force} = m_1 \times m_2 / r_{12}^3$$

- The moon dominates the tide. The much greater mass of the sun than the moon is compensated for by the much greater distance to the sun (and this distance is cubed).
- Therefore the sun's tide generating force is about 46% of that of the moon.

B. The Barycenter

- The Earth and the moon actually orbit the sun together as a system. They rotate around the center of the Earth-moon system, which is called the *barycenter*.
- The barycenter is actually within the Earth's mantle because the Earth is so much larger than the moon...it's about 4700 km from the center of the earth, which puts it in the lithosphere

C. Centripetal Force

- To understand the tide-generating force you need to understand *centripetal force*.
- The Centripetal force that keeps the Earth-moon system together is provided by gravity. All particles of the Earth follow circles of equal radius as they rotate in the Earth-moon system.
- The centripetal force required to keep each particle in an identical orbit is the same and is directed toward the center of the orbit.
- While the average gravitation attraction must equal the average centripetal force, these two are not equal for all points on Earth.

- Unlike the centripetal force, which is the same for all particles, the gravitational attraction of the moon (which supplies the centripetal force) is greater for particles closer to the moon and is always directed toward the center of the moon.
- The tides result from the residual force which is determined by the vector subtraction of the gravitational attraction and centripetal force.
- These tiny lateral forces generate the bulges at the opposite sides of the Earth.
- These small forces decrease with the cube of the distance from Earth, so other planets and stars have less influence than the moon, which is much closer to the Earth.

III. Equilibrium Theory of Tides

A. Equilibrium Theory

- The easiest way to understand the tides is to imagine two tidal bulges, one toward and one away from the moon as it rotates on its axis
- If we then make 3 critical assumptions:
 1. The oceans are uniformly deep
 2. There is no friction between the oceans and the basins
 3. There are no continents
- We get the *Equilibrium Theory of Tides*
- this is the “ideal” tide cycle, and doesn’t really exist!

B. Lunar Cycles

- Since the Earth rotates once every 24 hours, the period between high tides - the *tidal period* - should be 12 hours (when the moon lines up directly overhead a point, and when it is exactly opposite on the other side of the earth)
- However, the moon continues to move around the Earth as the Earth rotates on its axis.
- Therefore, the high tide occurs every 12 hours and 25 minutes (the tidal period), according to the *lunar day*, which is 24 hours and 50 minutes long.

C. Spring and Neap Tides

- There are combined effects of the sun and moon.
- The moon goes through all its phases about every 29.5 days. The new moon and the full moon occur when the moon is aligned with the sun.
- The tide generating forces of the sun and moon then add together by constructive interference.
- This results in the maximum tidal range and produces **spring tides** (which has nothing to do with the season!)
- The quarter moon results when the moon is at right angles to the Earth-Sun axis.
- This results in destructive interference and the tidal range is minimal producing **neap tides**.
- There is about two weeks between successive spring tides or successive neap tides.

D. Declination and Ellipses

- The moon and the sun are not over the Earth's equator for most of the year, but are north or south of it.
- This angular distance from the Earth's equatorial plane is called the *declination*.
- The amount of this declination changes with time over a regular cycle.
- The plane in space which includes the ellipse of the Earth's orbit is known as the *ecliptic*.
- The Earth leans by an angle of 23.5 degrees to the ecliptic, as we have seen earlier in the course.
- This tilt of the Earth, which always points in the same direction, leads to the seasons. At the vernal equinox, around March 21, the sun is directly over the equator. The greatest northern movement of the sun (overhead at the Tropic of Cancer, 23.5 degrees N) is about June 21, at the summer solstice. The sun is again overhead at the equator around Sept. 23 for the autumnal equinox. It reaches its southernmost extent about Dec. 22 at the winter solstice, where it is overhead above the Tropic of Capricorn at 23.5 degrees S.
- The orbit of the Earth around the sun and the orbit of the moon around the Earth are both elliptical. The tide generating forces of

the moon and sun are greatest when they are closest to the Earth in these elliptical orbits.

- The plane of the moon's orbit also rotates slowly, about every 18.6 years...this is called *precession*
- To summarize, the orbital paths of the Earth-moon system result in changes in the ideal tides because of:
 1. Constructive/Destructive interference with the Sun
 2. The moon's declination
 3. The Earth's ecliptic and tilt
 4. The moon's precession

IV. Dynamical Theory of Tides

A. Other issues

- In addition to the changes in orbital paths, we must also account for the friction of the oceans, and the fact that there are continents
- Since tides are shallow water waves, their speed is limited by the mean depth of the ocean to about 700 kilometers/hour (435 miles/hour).
- The tidal bulges are also limited by the location of the continents which restrict their movement.
- Therefore, instead of two bulges the tides break up into numerous cells...these are influenced by reflection, refraction, and diffraction interactions with the continents

B. Dynamical Theory

- When we combine all of these interactions, we find that there are rotating cells around which the tidal waves move.
- The center of the cell is called an *amphidromic point*, and there is no tide at all there, because it's the center of rotation.
- Radiating out from the amphidromic point are *cotidal lines* along which the high (and low) tides occur simultaneously.
- The tide wave rotates around the amphidromic point counterclockwise in the Northern Hemisphere and clockwise in the Southern Hemisphere and makes one complete rotation during the tidal period (about 12 hours)

- These move the opposite direction of the coriolis force because the tides are such large waves that they turn into standing waves...the crest rotates counterclockwise, but the current itself is rotating clockwise, as we expect.

C. Types of Tides

- Relative to the types of tides, a theory would predict two high tides and two low tides of unequal height during a lunar day.
- However, due to the varying shapes, sizes, and depths of the ocean basins, there are three different patterns of tides.

These include:

- 1) **Diurnal tides** (daily) - One single high and low tide each day, the common type found in the Gulf of Mexico.
- 2) **Semidiurnal tides** (twice daily) - Two high and two low tides each lunar day. The two highs and two lows are about the same height. These are found along the U.S. Atlantic coast.
- 3) **Mixed tides** - These have characteristics of both the other types. Successive high (and low) tides are of different heights. They are most commonly semidiurnal with a tidal period of 12 hours and 25 minutes, but they may have diurnal periods for a few days each month. They are common along the U.S. Pacific coast.